INTRODUCTION TO HYDROLOGIC MODELING

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1.0 Introduction

Rainfall-runoff modeling is an important aspect of hydrologic analysis and design. Choice of an appropriate approach to modeling of rainfall to runoff transformation process in a basin is influenced by various factors which include (i) typical features of the system; (ii) objectives of the study; (iii) degree of realism; and (iv) availability of data and resources; and (v) time scale of analysis.

A study of a river basin's hydrology and consequently, the design of suitable study approach, by its very nature is quite extensive. While the design of suitable hydrologic methodologies is an important consideration equally critical is the manner of application of these methodologies. It is impossible to build exact scale models of the hydro-climatological systems on which one could perform experiments to understand the nature of its operations on rainfall and its eventual transformation into runoff. The mathematical modeling approach is an alternative path. Evolution of the art of mathematical modeling of basin hydrology, often used for the purpose of runoff simulation, has followed several courses. We first explain here the rainfall-runoff process.

2.0 Rainfall-Runoff Models

A model is a simplified representation of a complex system. It aids in making decisions, particularly where data or information are scarce or there are large-number of options to choose from. Hydrological models represent the physical/ chemical/biological characteristics of the catchment and simulate the natural hydrological processes. Hydrological models are essentially mathematical models where the physical processes of hydrologic cycle are described by a set of mathematical equations, logical statements, boundary conditions and initial conditions, expressing relationships between inputs, variables and parameters. Hydrological models may be broadly classified in two groups:

(i) Deterministic Hydrological Models,
(ii) Stochastic Hydrological Models.

A deterministic hydrological model is one in which the processes are modelled based on definite physical laws and no uncertainties in prediction are admitted. Deterministic models permit only one outcome from a simulation with one set of inputs and parameter values. It has no component with stochastic behaviour, i.e. the variables are free from random variation and have no distribution in probability. Deterministic models can be further classified according to whether the model gives a spatially lumped or distributed description of the catchment area, and whether the description of the hydrological processes is empirical, conceptual or fully physically based.

Stochastic models allow for some randomness or uncertainty in the possible outcomes due to uncertainty in input variables, boundary conditions or model parameters. The vast majority of models used in rainfall-runoff modelling are used in a deterministic way, although again the distinction is not clear-cut since there are examples of models which add a stochastic error model to the deterministic predictions of the hydrological model and there are models that use a probability distribution function of state variables but make predictions in a deterministic way. A working rule is that if the model output variables are associated with some variance or other measure of predictive dispersion the model can be considered stochastic; if the output values are single valued at any time step the model can be considered deterministic, regardless of the nature of the underlying calculations.

Empirical or black box models contain no physically based transfer function to relate input to output. In other words no consideration of the physical processes is involved in such models. These models are basically input-output based models. Within the range of calibration, such models may be highly successful. However, in extrapolating beyond the range of calibration, the physical link is lost and the prediction relies on mathematical technique alone.

Lumped conceptual models occupy an intermediate position between the fully distributed physically based approach and empirical black box analysis. Lumped models treat the catchment as a single unit, with state variables that represent averages over the catchment area, such as average storage in the saturated zone. Such models are formulated on the basis of a relatively small number of components, each of which is a simplified representation of the process element in the system being modelled. Parameters of such type of models are calibrated using trial and error method or automatic optimisation technique or combination of both.

Fully distributed physically based models are based on our understanding of the physics of the hydrological processes which control catchment response and use physically based equation to describe these processes. From their physical basis such models can simulate the complete runoff regime, providing multiple outputs (e.g. river discharge, phreatic surface level and evaporation loss) while black box models can offer only one output. Unlike lumped conceptual models, physically based distributed models do not consider the transfer of water in a catchment to take place between a few defined storages. Instead the transfers of mass, momentum and energy are calculated directly from the governing partial differential equations.

There is a general correspondence between lumped models and the 'explicit soil moisture accounting' (ESMA) models and between distributed models and 'physically based' or process-based models. Even this correspondence is not exact, however, since some distributed models use ESMA components to represent different sub-catchments or parts of the landscape as hydrological response units, while even the most distributed models currently available must use average variables and parameters at grid or element scales greater than the scale of variation of the processes. They are consequently, in a sense, lumped conceptual models at the element scale. There is also a range of models that do not make calculations for every point in the catchment but for a distribution function of characteristics.

A question can arise at this stage as to how choose a particular model structure for a particular application? The following procedure is suggested based, in essence, on considerations of the function of possible modelling structures:

1. Prepare a list of the models under consideration. This list may have two parts: those models that are readily available, and those that might be considered for a project if the investment of time (and money!) appeared to be worthwhile.

2. Prepare a list of the variables predicted by each model and those required. Decide whether the model under consideration will produce the outputs needed to meet the aims of a particular project. If you are interested in the rise in the water table in valley bottoms due to deforestation, for example, a model predicting the lumped response of the catchment may not fulfill the needs of the project. If, however, you are only interested in predicting the discharge response of a catchment for real-time flood forecasting, then it may not be necessary to choose a distributed modelling strategy.
3. Prepare a list of the assumptions made by the model (see the guides in the chapters that follow). Are the assumptions likely to be limiting in terms of what you know about the response of the catchment you are interested in? Unfortunately the answer is likely to be yes for all models, so this assessment will generally be a relative one, or at best a screen to reject those models that are obviously based on incorrect representations of the catchment processes.
4. Make a list of the inputs required by the model, for specification of the flow domain, for the specification of the boundary and initial conditions and for the specification of the parameter values. Decide whether all the information required can be provided within the time and cost constraints of a project.
5. Determine whether you have any models left on your list. If not, review the three previous steps, relaxing the criteria used. If predictions are really required for an application, one model at least will need to be retained at this stage!

3.0 Rainfall Runoff Modelling

A brief description of important model types is given here.

3.1 The Rational Method

Rainfall-runoff modelling has a long history and the first hydrologists attempting to predict the flows that could be expected from a rainfall event were had insight into hydrological processes, even if their methods were limited by the data and computational techniques available to them. We can go back nearly 150 years to the first widely used rainfall-runoff model, that of the Irish engineer Thomas James Mulvaney (1822-1892) and published in 1851. The model was a simple equation that manages to illustrate most of the problems that are associated with hydrological modeling. The equation was:

$$Q_p = CAR_m \tag{1}$$

The Mulvaney equation (also known as the rational formula) does not attempt to predict the whole hydrograph but only the hydrograph peak Q_p. This is often all an engineering hydrologist might need to design a bridge or culvert capable of carrying the estimated peak discharge. The input variables are the catchment area, A, a maximum catchment average rainfall intensity, R_m, and an empirical coefficient or parameter, C. Thus, this model reflects the way in which discharges are expected to increase with area and rainfall intensity in a rational way. In fact, variations on equation (3) were published by a variety of authors based on different empirical data sets and are still in use today.

The scaling parameter C reflects the fact that not all the rainfall becomes discharge, but here the method is not quite so rational since it makes no attempt to separate the different effects of runoff production and runoff routing that will control the relationship between the volume of rainfall falling on the catchment in a storm, effectively AR_m, and the discharge at the hydrograph peak. In addition, the coefficient C is required to take account of the nonlinear relationship between antecedent conditions and the profile of storm rainfall and the resulting runoff production. Thus C is not a constant parameter, but will vary from storm to storm on the same catchment, and from catchment to catchment for similar storms. The easiest way to get a value for C is to back-calculate it from observations of rainfall and peak discharge (the very simplest form of model calibration). Predicting the correct value for a different set of conditions, perhaps more extreme than those that have occurred before or for a catchment that has no observations, is a much more difficult task.

Similar difficulties persist to the present day, even in the most sophisticated computer models. It is still difficult to take proper account of the nonlinearities of the runoff production process, particularly in situations where data are very limited. It is still easiest to obtain effective parameter values by back-calculation or calibration where observations are available; it remains much more difficult to predict the effective values for a more extreme storm or ungauged catchment. There are still problems of separating out the effects of runoff production and routing in model parameterizations (and in fact this should be expected because of the real physical interactions in the catchment).

3.2 Black box and stochastic models

Black box models attempt to develop empirically identified statistical relationships between rainfall and runoff, without attempting to define and understand the physical processes invoked in the transformation. Generally, these methods require fitting of a mean line through the scatter of plot of runoff against rainfall and use of this mean line or the curve drawn through the points to predict the runoff associated with a particular rainfall. While these models are generally simple to use, they may not be appropriate models of basins which are highly regulated and still under development.

Another class of black box type rainfall-runoff models are the non-parametric Unit Hydrograph based models. These models seek a linear, time invariant and casual relationship between rainfall excess and direct runoff. A variation of this approach is the Linear Perturbation Model (Nash and Barsi, 1983). This class of models establishes a non-parametric linear relationship between total rainfall and total runoff and occupies a special place amongst the group of models called Total Response Linear Models.

Apart from these three model classes, time series methods have also been widely used to design stochastic models of river flow. An essential requirement for the success of this approach is that the processes being modeled must be stationary. This is because these models attempt to preserve the serial correlations in the series being modeled. Where non-stationarity in river flow is principally due to non-stationarity in climatic influences, e.g., rainfall, one could, at least theoretically, use a Transfer Function Noise, TFN, type stochastic model of river flows, in which rainfall is used as an exogenous predictor. Intervention models have also been proposed to incorporate and understand the impact of human influences on river flow. While it may be argued, this class of stochastic models has physical justification, the understanding of water resources utilization within the river basin and the associated elegance of modeling, as permitted by a formal and an explicit water balance approach, will be missing in this approach.

In the context of rainfall-runoff modeling, stochastic models lack the intuitive appeal because these models do not imply the cause and effect relationship between the input and output variables that exists in a typical hydrologic system and have been, with a few exceptions, generally avoided for rainfall-runoff modeling (Todini, 1988). Research in the area of Artificial Intelligence has also resulted in application of techniques based on Artificial Neural Networks to the problem of modeling rainfall to runoff transformation processes and may be classified as a black box model.

3.3 Regression Models

Assuming that a general linear relationship with a memory length *m* exists between rainfall P and runoff Q, runoff may be expressed as:

$$Q_i = a_1 P_i + a_2 P_{i-1} + \ldots\ldots + a_m P_{i-m+1} + u_i \qquad \ldots(2)$$

where a_i = regression coefficients, and u_i = the error term. Vector of a values, which are unknown, is estimated by method of least-squares by minimizing the sum of error squares. After obtaining the regression coefficients a, the Q values can be obtained using the above equation.

Monthly rainfall-runoff relationship for gauged catchments

In India more than 80% of the annual rainfall is received in monsoon season (normally from June to October). The rainfall-runoff relationships for monsoon months may be developed using linear rainfall-runoff model. However, during non-monsoon months (Nov-May) most of the runoff in the stream is due to contribution of the ground water reservoir (base flow). The contribution of the rainfall is almost negligible except that a few thunder storms may contribute to streamflow. For partially snow fed basins, snow melt runoff constitutes a significant part of the streamflow. While developing the monthly rainfall runoff relationships, it is necessary to identify the monsoon months for the study area as well as type of the basin i.e. snow fed or rain fed. If the basin is partially snow fed and partially rainfed, monthly snow water equivalents are needed in addition to monthly rainfall data.

3.4 Unit Hydrograph

The unit hydrograph (UH) theory proposed by Sherman (1932) is primarily based on the principle of linearity and time and space invariance. A UH is a hydrograph of surface runoff resulting at a given location on a stream from a unit rainfall excess amount occurring in unit time uniformly over the catchment area up to that location. The rainfall excess excludes losses (abstractions) from total rainfall and unit rainfall excess normally equals 1 mm. The selection of unit time depends on the duration of storm and size of the catchment area. For small catchments, periods of 1 or 2 hours can be assumed and for larger catchments, 3, 4, 6, or even 12 hours can be adopted. Thus

(i) A UH is a flow hydrograph;
(ii) A UH is a hydrograph of direct surface runoff (DSRO), not total runoff;
(iii) The hydrograph of surface runoff results from the rainfall excess;

(iv) The rainfall excess represents total rainfall minus losses (abstractions);

(v) During the unit time period, the rainfall excess is assumed to occur uniformly over the catchment;

(vi) Typical unit times used in UH analyses are 1, 2, 3, 6, 8, and 12 hours. Beyond this, time period is generally taken as an integer multiple of 24 hours.

A unit hydrograph can be interpreted as a multiplier that converts rainfall excess to direct surface runoff. The direct surface runoff (DSRO) is the streamflow hydrograph excluding baseflow contribution. Since, a unit hydrograph depicts the time distribution of flows, its multiplying effect varies with time. In real-world application, the unit hydrograph is applied to each block of rainfall excess and the resulting hydrographs from each block are added for computing direct surface runoff hydrographs, to which baseflows are further added to obtain total hydrographs.

Factors Affecting UH Shape

The factors affecting the shape of the unit hydrograph are the rainfall distribution over the catchment and physiography of the catchment, viz., shape, slope, vegetation, soil type, etc. Variations in areal pattern of rainfall, rainfall duration, and time intensity pattern greatly affects the shape of the hydrograph. A hydrograph resulting from rainfall concentrated in the lower part of a basin will exhibit a rapid rise, sharp peak, and rapid recession. On the other hand, rainfall concentrated in the upper part of the same basin will yield a slow rising and receding hydrograph having broad peak. Thus, UHs developed from rainfall of different areal distributions will exhibit differing shapes. Given the amount of runoff, the time base of the unit hydrograph increases and peak lowers as the duration of rainfall increases.

Natural physical characteristics of a watershed are affected by man's influence, for example, follow-up of watershed management practices significantly change the land cover, and consequently, the shape of the derived UH also changes. Steep catchment slopes produce runoff peak earlier than flatter slopes. Consequently, UHs of steep catchments exhibit peaks occurring earlier than those of flatter slopes. Urbanization of a catchment causes drastic changes in the shape of hydrograph and, in turn, the UH. Urbanisation reduces the natural storage of the basin as well as the average loss rate. As a result, the derived UH exhibits higher peak and shorter time of concentration. Seasonal and long-term changes in vegetation or other causes, such as fire, also changes the physical characteristics of the watershed. It resorts to developing a regional relationship between UH parameters and existing basin characteristics, for deriving the unit hydrograph in the changed environment.

4.0 Conceptual models

These models occupy an intermediate position between the fully physically-based approach and empirical black-box analysis. Such models are formulated on the basis of a relatively small number of components, each of which is a simplified representation of one process element in the system being modeled. Models belonging to this group describe catchments as storages which are connected according to a defined rule. Nash Cascade Model, Stanford Watershed Model (SWM), Sacramento model, and Tank Model are some well known conceptual models. In the Tank model model, the catchment is represented by a series of tanks. Recently, Todini (1996) introduced the ARNO rainfall-runoff model. This is a semi-distributed conceptual model using spatial probability distribution of soil moisture capacity and dynamically varying saturated contributing areas. A detailed treatment of lumped models is given by *Blackie and Eeles* [1985] and Singh (1995).

4.1 SCS Curve Number Method

The Soil Conservation Service (SCS) of USA has developed a procedure for estimating runoff from small watersheds. This empirical procedure was developed to provide a rational basis for estimating the effects of land treatment and land use changes upon runoff resulting from storm rainfall. Because of its simplicity, however, it has been widely used by agriculturists, hydrologists and soil conservation engineers.

The SCS Curve Number method is widely used because (i) it is a reliable procedure that has been used for many years in different parts of the world, (ii) it is computationally efficient, (iii) the required inputs are generally available, and (iv) it relates runoff to soil type, land use and management practices.

The volume of runoff depends on both meteorologic and watershed characteristics. The precipitation volume is the single most important meteorological characteristics in estimating the runoff. The soil type, land use and the hydrologic condition of the cover are the watershed factors that will have significant effect on the volume of runoff.

The SCS developed an index, which is called the runoff curve number (CN) to represent the combined hydrologic effect of soil, land use, agricultural land treatment class, hydrologic condition and antecedent soil moisture. The curve number, CN, is directly used in the relationship to estimate the runoff.

The volume of runoff (Q) depends on the volume of precipitation (P) and the volume of storage that is available for retention. The actual retention (F) is the difference between the volumes of precipitation and runoff. The main assumptions of the SCS-CN method are: i) runoff begins after an initial abstraction I_a consisting of interception, surface storage, and infiltration has been satisfied, and ii) The ratio of actual retention of rainfall to the potential maximum retention S is equal to the ratio of direct runoff to rainfall minus initial abstraction. Thus

$$(P - I_a - Q) / S = Q / (P - I_a) \tag{3}$$

or
$$Q = (P - I_a)^2 / [(P - I_a) + S] \tag{4}$$

The initial abstraction is a function of land use, treatment, and condition; interception; infiltration; depression storage; and antecedent soil moisture. An empirical analysis was performed by SCS and it was found that 20% of the potential maximum retention is the initial abstraction before runoff starts, or $I_a = 0.2S$. Therefore,

$$Q = (P - 0.2S)^2 / (P + 0.8S) \tag{5}$$

Recent research suggests that the relation $I_a = 0.2S$ may not be correct under all circumstances. However, it remains in use until more reliable results are available. Note that it implies that the factors affecting I_a could also affect S. The parameter S depends upon the soil, vegetation, land use and antecedent moisture condition (AMC) of a catchment. The SCS expressed S as a function of a parameter termed *curve number* :

$$S = 1000 / CN - 10 \quad (6)$$

where CN is the runoff curve number. It represents a measure of retention of water by a soil-vegetation-land use complex. Its permissible range is 0-100. Since S is a function of the factors that affect I_a one can expect that the CN would also be a function of land use, antecedent soil moisture, and other factors that affect runoff and retention. Note that when S is zero, CN is 100 and this leads to Q = P.

To determine the parameter CN, the soil type is divided in four groups. The AMC which represents the moisture content of the soil at a given time has been classified in three groups: AMC I - dry soil, AMC II - medium conditions, and AMC III - saturated soil.

SCS Soil Group Classification

SCS developed a soil classification system that consists of four groups, which are identified by the letters A, B, C and D. Soil characteristics that are associated with each group are as follows:

Group A:	deep sand, deep loess, aggregated silts,
Group B:	shallow loess, sandy loam,
Group C:	clay loams, shallow sandy loam, soils low in organic content, and soils usually high in clay,
Group D:	soils that swell significantly when wet, heavy plastic clays, and certain saline soils.

SCS Cover Complex Classification

The SCS cover complex classification consists of three factors: land use, treatment or practice, and hydrologic condition. For estimating the curve numbers, approximately fifteen different land uses have been identified. Agricultural land uses are often subdivided by treatment or practices, such as contoured or straight row; this separation reflects the different hydrologic runoff potential that is associated with variation in land treatment. The hydrologic condition reflects the level of land management; it is separated with three classes: poor, fair and good. Not all of the land uses are separated by treatment or condition.

Antecedent Soil Moisture Condition

Antecedent soil moisture is known to have a significant effect on both the volume and rate of runoff. Recognizing that it is a significant factor, SCS developed three antecedent soil moisture conditions, which were labelled I, II and III. The soil conditions for each are as follows:

I:	Soils are dry but not to wilting point, satisfactory cultivation has taken place.
II:	Average conditions.
III:	Heavy rainfall, or light rainfall and low temperatures have occurred within the last 5 days; saturated soil.

The following table gives seasonal rainfall limits for the three antecedent soil moisture conditions:

	Total 5-day Antecedent Rainfall (cm)	
AMC	Dormant Season	Growing Season
I	Less than 1.27	Less than 3.56
II	1.27 to 2.73	3.56 to 5.33
III	Over 2.73	Over 5.33

In design, the antecedent soil moisture condition is often a policy decision rather than a statement of actual soil condition at the site during development.

5.0 Physically-based Distributed Models

Now-a-days engineers, scientists and planners involved in water resources development have become more concerned with the effect of land use changes related to agricultural and forestry practices, hazards of pollution and toxic waste disposal and general problem arising from conjunctive uses of water. Conventional rainfall runoff models (empirical as well as lumped conceptual models) are often not able to provide satisfactory solutions to such problems. Attention is, therefore, being focused on the physically based distributed catchment models since these have the potential to overcome many of the deficiencies associated with simpler approaches. On the other hand, such models are complex and considerable resources in human expertise and computing capability are needed for their development and applications. Distributed models make predictions that are distributed in space, with state variables that represent local averages of storage, flow depths or hydraulic potential, by discretizing the catchment into a large number of elements or grid squares and solving the equations for the state variables associated with every element grid square. Parameter values must also be specified for every element in a distributed model.

These models simulate or mimic the hydrological processes governing the relationship between rainfall and runoff. Well known examples of this class of models are the European Hydrological System - Systeme Hydrologique European or SHE model (Abbott et al., 1986) and IHDM (Beven 1987). The SHE system can model all or any part of the land phase of the hydrological cycle. Based, as process models are, on complex physical theory, a high degree of physical realism is sought to be achieved in this approach. An important advantage of developing and using physics based deterministic models is their perceived value in helping to improve our understanding of the hydrologic system. For this reason these models are appropriate tools for modeling dynamics of hydrologic systems in which major changes in land use, in drainage or river control systems occur (Blackie and Eeles, 1985). Jain et al. (1992) applied to SHE model to a few catchments in India.

Beven (1989) wrote a detailed treatise on the scope of physically based models. The paper provides an excellent insight into the as yet unresolved, problems that bedevil this approach. While stating that the theoretical advantages of physically based models remain unproven, the author in his critical evaluation of the physics based approach to hydrologic modeling highlights the following serious drawbacks in this approach. According to Beven (1989), ‘the physics on which the equations are based is the small scale physics of homogeneous systems’. In actual applications, lumping of the small scale physics to the model grid scale is required to be done for a numerical solution to the transport equations. There is, however, no theoretical framework for carrying out this lumping of subgrid processes for spatially heterogeneous grid squares. It is merely assumed that the same small scale physics equations can be applied at the model grid scale with the same parameters. This, according to the author, is making a leap into the realm of the conceptual approach. Beven (1989) asserts that it is easy to demonstrate the conceptual nature of current physically based models and states that '… results of the implicit lumping of subgrid processes inherent in physically based models. We cannot be sure that the equations will be the same at the grid scale, nor whether effective grid scale parameters can be defined. Preliminary studies on lumping of subgrid processes in hydrology seem to suggest that it is not possible to define a consistent effective parameter value to reproduce the response of a spatially variable pattern of parameter values.

5.1 The Soil and Water Assessment Tool (SWAT) Model

SWAT is a spatially distributed, continuous time scale watershed model developed by Dr. Jeff Arnold for the USDA-ARS. It was developed to predict the impact of land management practices on water, sediment and agricultural chemical yields in large complex watersheds with varying soils, land use and management conditions over long periods of time. Weather, soil properties, topography, vegetation and land management practices are the most important inputs for SWAT to model hydrologic and water quality in a watershed (Neitsch, 2002).

SWAT model is comprised of numerous diverse physical processes in the basin to be modeled. Catchment has to be divided into sub-catchments for the purpose of modelling. Sub-catchment division in simulations is very useful in the environment with catchment parts having significantly different characteristics of vegetation or soil that has an impact on hydrologic processes. Division of basic catchment areas within the sub-catchments allows the user to distinguish between relevant catchment areas and analyze them. Input data for each sub catchment are grouped or organized into the following categories: climate, HRUs, storages/lakes, underground, river network and catchment runoff. Regardless of the type of problem being modeled and analyzed by the model, background of the method is the water balance of the catchment area. In order to achieve precise forecast of circulation of the pesticides, sediments or nutrients, hydrologic cycle is simulated by the model which integrates overall water circulation in the catchment area. Hydrologic simulations in the catchment area can be divided into two groups. In the soil phase of the hydrologic cycle, the processes on the surface and in the sub-surface soil occur, as well as the circulation of sediments, nutrients and pesticides through the water flows in all sub-catchments. In the second phase, the circulation of water and sediment through the river network up to the exit profile are observed.

SWAT is a semi-distributed, continuous watershed modelling system, which simulates different hydrologic responses using process- based equations. The model computes the water balance from a range of hydrologic processes such as evapotranspiration, snow accumulation, snowmelt, infiltration and generation of surface and subsurface flow components. Spatial variability within a watershed is represented by dividing the area into multiple sub-watersheds, which are further subdivided into hydrologic response units (HRUs) based on soil, land cover and slope characteristics.

SWAT uses a temperature-index approach to estimate snow accumulation and melt. Snowmelt is calculated as a linear function of the difference between average snowpack maximum temperature and threshold temperature for snowmelt. Snowmelt is included with rainfall in the calculation of infiltration and runoff. SWAT does not include an explicit module to handle snow melt processes in the frozen soil, but includes a provision for adjusting infiltration and estimating runoff when the soil is frozen (Neitsch et al., 2005). Despite this limitation, SWAT is considered to be an appropriate integrated model for addressing a range of issues.

6.0 Model Calibration and Validation

Once one or more models have been chosen for consideration in a project, it is necessary to address the problem of parameter calibration. Unfortunately, it is not, in general, possible to estimate the parameters of models by either measurement or prior estimation. Studies that have attempted to do so have generally found that, even using intensive series of measurements of parameter values, the results have not been entirely satisfactory. Prior estimation of feasible ranges of parameters also often results in ranges of predictions that are wide and may still not encompass the measured responses all of the time.

The model performance is typically evaluated from the comparison of simulated and the observed discharge data in terms of mean, standard deviation, maximum daily discharge, and the total discharge using commonly used indices. To evaluate the model performance, three statistical indices are commonly used to evaluate the model performance: Coefficient of determination (R^2), Nash and Sutcliffe efficiency (E), index of agreement (d), and relative error of the stream flow volume (RE). These are computed as follows.

Coefficient of determination (R^2): Willmott (1981) and Leagates and McCabe Jr. (1999):

$$R^2 = \left(\frac{\sum_{i=0}^{n} \left(Y_i^{obs} - Y_{mean}^{obs}\right)\left(Y_i^{sim} - Y_{mean}^{sim}\right)}{\sqrt{\sum_{i=1}^{n} \left(Y_i^{obs} - Y_{mean}^{obs}\right)^2} \sqrt{\sum_{i=1}^{n} \left(Y_i^{sim} - Y_{mean}^{sim}\right)^2}} \right)^2 \qquad \ldots.(7)$$

Nash-Sutcliffe coefficient (E_{NS}): Nash and Sutcliffe (1970) developed this index:

$$E_{NS} = 1 - \left[\frac{\sum_{i=1}^{n} \left(Y_i^{obs} - Y_i^{sim}\right)^2}{\sum_{i=1}^{n} \left(Y_i^{obs} - Y_{mean}^{obs}\right)^2} \right] \qquad \ldots.(8)$$

Index of agreement (d): It was proposed by Willmott (1981) as:

$$d = 1 - \frac{\sum_{i=1}^{n} \left(Y_i^{obs} - Y_i^{sim}\right)^2}{\sum_{i=1}^{n} \left(\left|Y_i^{sim} - Y_{mean}^{obs}\right| + \left|Y_i^{obs} - Y_{mean}^{obs}\right| \right)^2} \qquad .(9)$$

RMSE-SD Ratio (RSR): Root Mean Square Error (RMSE) is a commonly used index to measure the performance of a model. However, its magnitude depends upon the range of values of the variable being examined. RSR is computed as the ratio of RMSE and the standard deviation (SD). By dividing by SD, the value of RMSE is standardized. RSR varies from optimal value of 0 (perfect model simulation) to a large positive value.

$$\mathrm{RSR} = \frac{\mathrm{RMSE}}{\mathrm{STDEV}_{obs}} = \frac{\left[\sqrt{\sum_{i=1}^{n}\left(Y_i^{obs} - Y_i^{sim}\right)^2}\right]}{\left[\sqrt{\sum_{i=1}^{n}\left(Y_i^{obs} - Y^{mean}\right)^2}\right]} \quad .(10)$$

To rate the performance of a model in calibration, Moriasi et al. (2007) recommended a range of values for RSR and NSE that can be used to categorize the performance of models. General performance ratings for recommended statistics for a simulation with monthly time step are given in Table 2.

Table 2: General performance ratings for recommended statistics for a simulation with monthly time step.

Performance Rating	RSR	NSE	PBIAS (%)	
			Streamflow	Sediment
Very good	$0.00 \leq \mathrm{RSR} \leq 0.50$	$0.75 < \mathrm{NSE} \leq 1.00$	$\mathrm{PBIAS} < \pm 10$	$\mathrm{PBIAS} < \pm 15$
Good	$0.50 < \mathrm{RSR} \leq 0.60$	$0.65 < \mathrm{NSE} \leq 0.75$	$\pm 10 \leq \mathrm{PBIAS} < \pm 15$	$\pm 15 \leq \mathrm{PBIAS} < \pm 30$
Satisfactory	$0.60 < \mathrm{RSR} \leq 0.70$	$0.50 < \mathrm{NSE} \leq 0.65$	$\pm 15 \leq \mathrm{PBIAS} < \pm 25$	$\pm 30 \leq \mathrm{PBIAS} < \pm 55$
Unsatisfactory	$\mathrm{RSR} > 0.70$	$\mathrm{NSE} \leq 0.50$	$\mathrm{PBIAS} \geq \pm 25$	$\mathrm{PBIAS} \geq \pm 55$

Source: Moriasi et al. (2007).

There are two major reasons for difficulties in calibration. The first is that the scale of the measurement techniques available is generally much less than the scales at which parameter values are required. For example, there may be a hydraulic conductivity parameter in a particular model structure. Techniques for measuring hydraulic conductivities of the soil generally integrate over areas of less than 1 m^2. However, even the most finely distributed models require values that effectively represent the response of an element with an area of 100 m^2 or a much larger area. For saturated flow, there have been some theoretical developments that suggest how such effective values might change with scale, given some underlying knowledge of the fine-scale structure of the conductivity values. In general, however, carrying out the experimental measurements required to use such a theory at the hillslope or catchment scale would be very time-consuming and expensive. Thus it may be necessary to accept that the small-scale values that it is possible to measure and the effective values required at the model element scale are different quantities (a technical word is that they are incommensurate) - even though the hydrologist has traditionally given them the same name. The effective parameter values for a particular model structure will then still need to be calibrated in some way.

Most calibration studies in the past have involved some form of optimization of the parameter values by comparing the results of repeated simulations with whatever observations of the catchment response are available. The parameter values are adjusted between each run of the model, either manually by the modeller or by some computerized optimization algorithm until some 'best fit' parameter set has been found. There have been many studies of different optimization algorithms and measures of goodness of fit or objective functions in hydrological modelling. The essence of the problem is to find the highest peak in the response surface in the parameter space defined by one or more objective functions.

It is highlighted here that the model structure and the observations are not error-free. Thus, the optimum parameter set found for a particular model structure may be sensitive both to small changes in the observations, or to the period of observations considered in the calibration, and possibly to changes in the model structure such as a change in the element discretization for a distributed model. There are a number of important implications that follow from these considerations:

- The parameter values determined by calibration are effectively valid only inside the model structure used in the calibration. It may not be appropriate to use those values in different models or in different catchments.
- The concept of an optimum parameter set may be ill-founded in hydrological modelling. While one optimum parameter set can often be found there will usually be many other parameter sets that are very nearly as good, perhaps from very different part in the parameter space. The idea of equifinality of parameters suggests that, given the limitations of both the model structures and observed data, there may be many representations of a catchment that may be equally valid in terms of their ability to produce acceptable simulations of the available data.

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